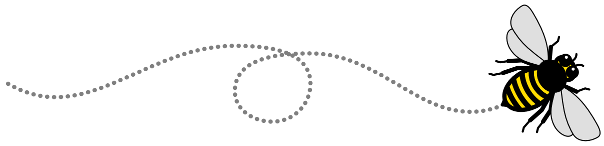
Bee-gon!



Question: Think about bees. What regular shape springs to mind?

Answer: A hexagon, right? Or maybe you had a different idea. In this activity we'll see why 'pentagon' is another good answer.

Part 1 – Family trees

Bee biology is quite different to human biology. Female bees have two parents: one male and one female. But male bees only have one female parent!

1. How many grandparents does a male bee have? How many great-grandparents does a male bee have?
2. Draw a family tree for a male bee which goes back 7 generations. To help you keep track, label each bee either M for male or F for female.
3. How many bee ancestors are there in each previous generation? Do you recognise this pattern? Can you explain to a classmate why it works?
4. Without drawing the family tree, how many ancestors are there 15 generations back?

Part 2 – Pentagons and pentagrams

For this activity you will need: a blank sheet of paper, a ruler, a protactor or geoliner, a sharp pencil and nerves of steel! Having trouble? Photocopy or print the pages provided.

5. Accurately draw a regular pentagon with side length 34 mm. What do you need to find out before you get started? Think about how the pentagon can be broken up into special triangles. What are the angles in those triangles?
6. Add the five diagonals to your pentagon to create a pentagram (5-pointed star). What new lengths, in millimetres, can you find in your diagram?
7. Now extend each side of your original pentagon in both directions to create a larger pentagram. Also join the points of the pentagram to create a larger pentagon. What new lengths can you find in your diagram?
8. If you've done this very accurately you should notice something! If not, try again with the pages provided. Or if you know about right-angled trigonometry, use any angles and lengths you know to find out other lengths without having to rely on a neat diagram.

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Part 3 – What's the connection?

Warning! More advanced material ahead.

When you square the number 3, the result is 9; instead, you could have got the same result by adding 6. When you square the number 2, you get the same result as adding 2. When you square the number 1, you've added 0.

Is there a number which, when squared, gives the same result as adding 1?

Yes! It's called the *golden ratio* and it is approximately equal to 1.618034. We use the Greek letter φ ('phi') to represent this number. So the defining property is $\varphi^2 = \varphi + 1$.

9. If you know about surds, verify that $\varphi = \dfrac{1+\sqrt{5}}{2}$ by substituting this expression into φ^2 and, after simplifying as much as possible, comparing it with $\varphi + 1$.

10. If you know about quadratic equations, show that $\varphi = \dfrac{1+\sqrt{5}}{2}$ is one of two solutions to the equation $x^2 = x + 1$. What is the other solution and how does it relate to φ?

The golden ratio and bees

Think about going back three generations, with X bees, Y bees and Z bees, respectively.

11. What is the relationship between X, Y and Z? Write down an equation.

12. Deduce that $\dfrac{Z}{Y} = \dfrac{X}{Y} + 1$.

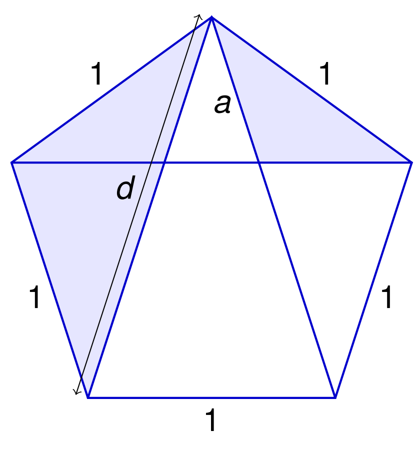
Eventually, when the numbers in each generation get large enough, the relative increase in size from one to the next stays roughly the same. (When you learn about sequences and limits, you can make this statement more precise.)

So, for example, if the generation with Y bees is roughly double the size of the one with X bees, then the generation with Z bees is also roughly double the size of the one with Y bees. That is, both increases would approximately be by a factor of 2. But the magic number here is *not* actually 2 – any ideas what it might be?

13. Assuming that the increases from X to Y and from Y to Z are by the same factor F, use the equation in Question 12 to deduce that F must equal the golden ratio!

The golden ratio and regular pentagons

Consider a regular pentagon with side length 1 and diagonals of length d, as shown.



14. Find all of the angles in the diagram.

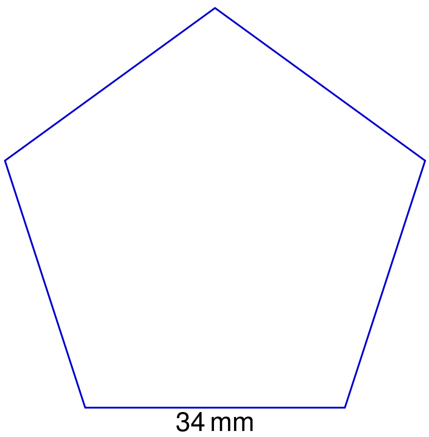
15. What is special about the two shaded triangles?

16. Explain why $1 + a = d$ and $ad = 1$. Now deduce that d is the golden ratio!

Can you work out why the sizes of each generation in a bee's family tree appear in the pentagrams formed from a pentagon with side length 34?

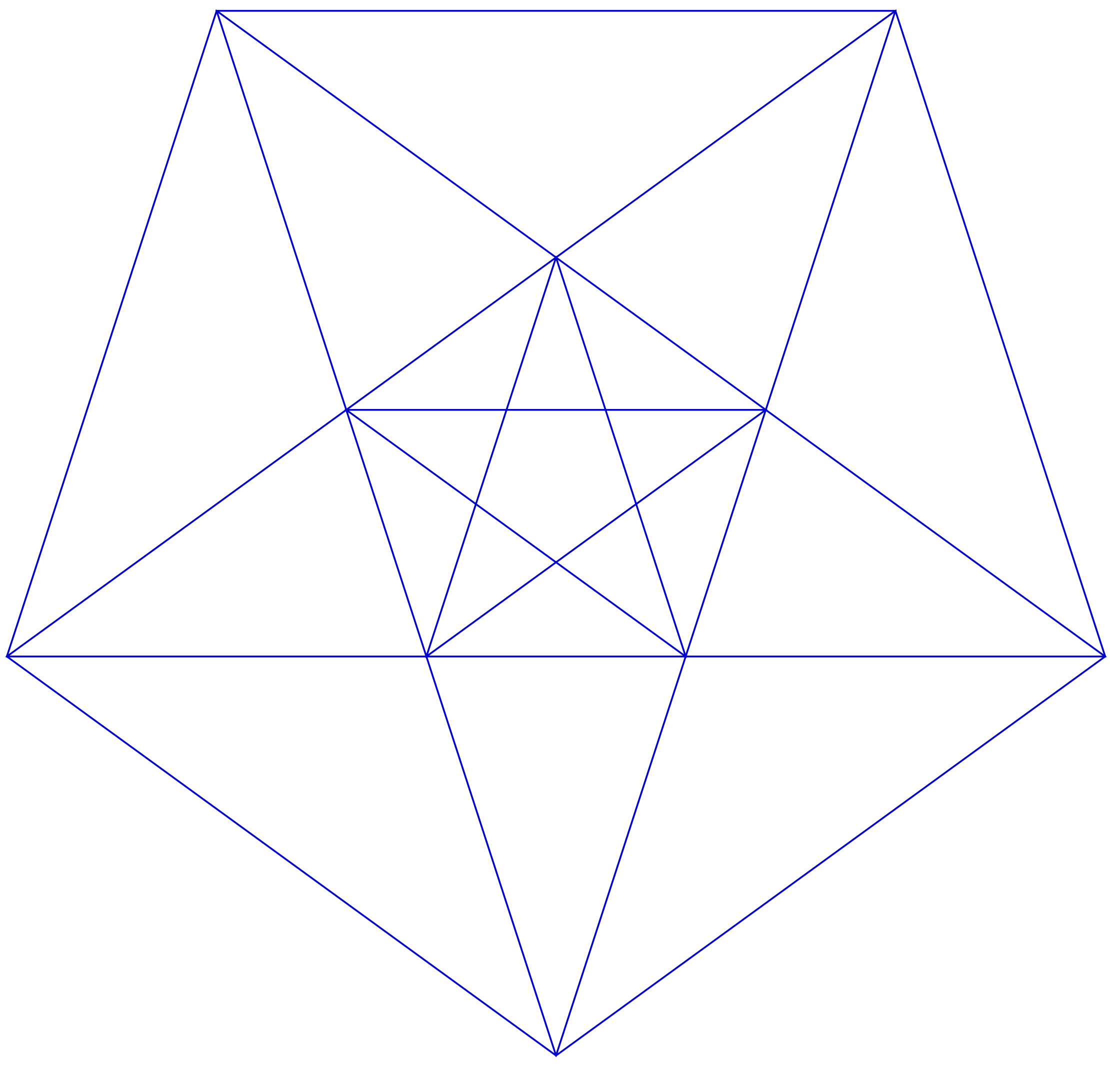
Having trouble drawing your own accurate pentagon?
Photocopy or print this page. Make sure you select ‘100%’ or ‘Actual size’.

Now add diagonals to create a pentagram on the inside. Also extend the sides to create a pentagram and new pentagon on the outside.



Having trouble adding accurate pentagrams to your pentagon?
Photocopy or print this page. Make sure you select ‘100%’ or ‘Actual size’.

What lengths, in millimetres, can you find in the diagram?



Answers below – do you recognise anything?

Note that these measurements are only approximate.
But it's still pretty convincing, right?